The RUDN University

Faculty of Physics, Mathematics and Natural Sciences

THE WORKING PROGRAM OF THE DISCIPLINE

Introduction to low-dimensional topology

Direction of training / specialty

01.04.01 Mathematics

(указываются код и наименование направления подготовки/специальности)

Profile

"Functional Methods in Differential Equations and Interdisciplinary Research"

(наименование образовательной программы в соответствии с направленностью (профилем)

1. Goals and objectives of the discipline: "Introduction to low-dimensional topology" are among the special disciplines that expand the professional horizons of the student-mathematics, contributing to his acquaintance with a number of both already classical and modern ideas that are in demand in many developing areas of mathematics, including, as a rule, in that of them, which is chosen by the student as his specialization.
The main goal of the course is to master the concepts and methods of the theory of classical and virtual nodes.

2. The place of discipline in the structure of OOP:
The discipline "Introduction to low-dimensional topology" refers to the discipline of the choice of the student of block 1 of the curriculum.

Table 1 shows the previous and subsequent disciplines aimed at the formation of discipline competencies in accordance with the competence matrix of EP HE

	Code and name of competence	Preceding disciplines	Subsequent disciplines (groups of disciplines)
	Профессиональные компетенции		
	PC.1. Able to conduct scientific research and obtain new scientific and applied results independently and as part of a research team	Modern problems of mathematics and applied mathematics, Non-Eucledean geometries and their applications	Undergraduate practice, State exam

3. Requirements for the results of mastering the discipline:

As a result of studying the discipline, the student must:

Know: Basic concepts of homotopy topology (cellular (CW) complex, fundamental group, homotopy groups, bundles, exact sequence, category, functor, smooth manifolds, etc.) and knot theory; statements of some basic problems of algebraic topology; formulations and ideas for proving a number of theorems present in the course program.

Be able to: Solve problems in all sections of the course at the level specified by the attached examples.

Own: The beginnings of each of the sections presented in the course.

4. Scope of discipline and types of educational work

The total workload of the discipline is 3 credit units.

№	**Type of educational work**	**Всего часов**	**Modules**			
			1	**2**	**3**	**4**
1.	**Auditory lessons**					36
1.1.	Lectures					18
1.2.	Other occupations					18
1.2.1	*Seminars)*					18
2.	**Independent work**					72
2.1.	Course work					
2.2.	Settlement and graphic works					

2.3.	Essay					
2.4.	Exam					
	Other types of independent work					72
3.	**Total (lessons)**					108
	Total (credits)					3

5. Discipline content

5.1. Contents of discipline sections

№	Section	Содержание раздела
1.	Homotopy methods in classical knot theory	Basic concepts of the theory of the classical theory of knots and links. Polynomial invariants of knots and links (Jones, Kaufman, Alexander polynomials, etc.) and their properties. Vasiliev invariants of finite order. Homotopy invariants of knots and links (knot group, techniques for calculating the fundamental groups of complements to knots and links).
2.	Homotopy methods in classical knot theory	Basic concepts of the theory of virtual knots and links. Polynomial invariants of virtual knots and links and their properties. Invariants of virtual links with values on graphs. Cooperberg bracket and "spiders". Elements of the theory of free knots. Parity bracket. Elements of the theory of braid groups

5.3. Sections of disciplines and types of classe

№	Section	Lectures	Seminars and laboratory works			IW	Total
			Sem.	Lab. works	of which interactively		
1.	Homotopy methods in classical knot theory	10	10			36	56
2.	Homotopy methods in virtual knot theory	8	8			36	52
	Total	18	18			72	108

6. Laboratory works

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7. Seminars

№	Section	Sem.
1	Homotopy methods in classical knot theory	10
2	Homotopy methods in virtual knot theory	8

	TOTAL	18

8. Course work

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9. Educational-methodical and informational support of the discipline:

a) Literature:

1. Милнор Дж., Уоллес А. «Дифференциальная топология. Начальный курс». М., «Мир», 1972.
2. Постников М.М. «Лекции по геометрии, семестр III. Гладкие многообразия». М., «Наука», 1987.
3. Прасолов В.В. «Элементы комбинаторной и дифференциальной топологии». М., МЦНМО, 2005.
4. Фукс Д.Б., Фоменко А.Т., Гутенмахер В.Л. «Гомотопическая топология». М. Изд-во МГУ, 1969.
5. Болтянский В.Г., Ефремович В.А. «Начальная топология». М., «Наука», 1982.
6. Дубровин Б.А., Новиков С.П., Фоменко А.Т. «Современная геометрия». М., «Наука», 1979.
7. Масси У., Столингс Дж. «Алгебраическая топология. Введение». М., «Мир», 1977.
8. Хирш М. «Дифференциальная топология». М., «Мир», 1979.
9. Ху Сы-Цзян. «Теория гомотопий». М., «Мир», 1964.

b) software: a package of typing and layout of mathematical texts TeX (for example, MikTeX 2.7), OpenOffice.org packages version not lower than 2.2, MSOffice version not lower than 2000, etc.

c) databases, reference and search systems:

provided by free Internet access in the teaching laboratories of the faculty and reading rooms of the RUDN University

10. Material and technical support of the discipline:

classroom for seminars, large audience (lecture hall) for lectures, laptop - 1 pc., projector - 1 pc., screen - 1 pc., copier - 1 pc., printer - 1 pc., scanner - 1 pc.

11. Guidelines for organizing the study of the discipline:

Compliance of grading systems (previously used grades of the final academic performance, ECTS grades and the point-rating system (BRS) of assessments of current performance) (In accordance with the Order of the Rector No. 996 dated December 27, 2006):

Points	Points of RF	Points	Points of RF	Points of ECTS
86 – 100	5	95 - 100	5+	A
		86 - 94	5	B
69 – 85	4	69 - 85	4	C
51 – 68	3	61 - 68	3+	D
		51 - 60	3	E
0 – 50	2	31 - 50	2+	FX
		0 - 30	2	F

1. Students are required to submit all assignments within the timeframe set by the teacher.
2. The point-rating system for assessing knowledge during the semester includes work in class, homework and study of the current material. 4 homework assignments are given on the topics indicated in the FOS, each of which is assessed from 10 points. A survey is conducted for the indicated sections, which is estimated at a maximum of 20 points.

3. The student is admitted to the final control with any number of points scored in the semester. The final control contains 2 tasks. One hour is given to prepare for the answer, after which the student is asked orally. A work of 50 points is evaluated regardless of the number of points received during the semester.
4. If, after the final control, the student received less than 31 points, then he is given an F mark and he must repeat the discipline in the prescribed manner. If, in the end, the student received at least 31 points, i.e. FX, then he is allowed to gain the required (up to 51) number of points by repeated one-time execution of the provided final control measures; at the same time, at the discretion of the teacher, the corresponding previous results are canceled. The liquidation of debts is carried out in the period from 07.02 to 28.02 (from 07.09 to 28.09) in agreement with the dean's office.

12. Fund of assessment tools for intermediate certification of students in the discipline (module) - attached.

The program is compiled in accordance with the requirements of OS 3 ++ RUDN University.

The developer

Ph.D., senior lecturer **V.A. Krasnov**

Director of the Mathematical Institute,
Doctor of Physical and

Mathematical Sciences, Professor **A.L. Skubachevskii**

Mathematical Institute

APPROVED BY
AT THE MEETING OF THE INSTITUTE
«___»_____20__ г., protocol №___
Director of Mathematical Institute
_____A.L. Skubachevskii
(подпись)

EVALUATION FUND

of discipline

Introduction to low-dimensional topology

01.04.01 Mathematics

Master
Квалификация (степень) выпускника

Discipline: Introduction to low-dimensional topology

Specialty: 01.04.01 Mathematics

Controlled competence code	Controlled discipline section	Supervised discipline theme	Name of the appraisal tool															Points of section	Points
			Current control											Intermediate certification					
			Interview	Test	Colloquium	Examination	Lab. works	Control works	Homework	Essay			⋮	⋮	Exam	⋮	⋮		
PC-1	Introduction to low-dimensional topology	Homotopy methods in classical knot theory				15			10						20			40	100
		Homotopy methods in virtual knot theory				15			10						20			40	
		TOTAL:				30			20						50			100	100

Appendix 3
(required)

Final control questions (2 questions, 25 points each)

1. Polygonal nodes and links. Isotopy of smooth knots. Reidemeister moves.
2. Examples of higher-order Vasiliev invariants.
3. The Kaufman bracket. Definition and examples of calculations.
4. Vasiliev invariants of order zero and first.
5. Invariance of the Kaufman bracket under the Reidemeister moves.
6. Derivative according to Vasiliev. Definition of Vasiliev invariants of finite orders.
7. Twist number and Jones-Kaufman polynomial. Examples of calculations.
8. The concept of singular (special) nodes.
9. Jones polynomial in one variable. Definition and examples of calculations.
10. A theorem on the transformation of a classical link diagram into a trivial link diagram by changing the types of some crossings.
11. Length of the Jones polynomial. Kaufman-Murasugi theorem.
12. Jones polynomial and the problem of recognition of mutant nodes.
13. Jones polynomial in connected sum and disjoint union of diagrams.
14. Structure of the Jones polynomial for links with an even (odd) number of components.
15. Virtual nodes.
16. Invariants of virtual nodes with values on graphs.

Tasks for control work 1

1. Calculate the Kauffman bracket for the classical knot diagram (5 points).
2. Calculate the Jones polynomial for the oriented diagram of the classical knot (10 points).

Tasks for control work 2

1. Calculate the Cooperberg bracket for the virtual diagram (5 points).
2. Check if the Kuperberg bracket is an invariant of virtual knots (10 points).

Option for assignments for home independent work

1. Prove the equivalence of two diagrams using Reidemeister's theorem (10 points).
2. For a given diagram, calculate the length of its Jones polynomial and, using this example, check the validity of the Kaufman-Murasugi theorem (10 points).