Federal State Autonomous Educational Institution of Higher Education

PEOPLES' FRIENDSHIP UNIVERSITY OF RUSSIA
RUDN University

Faculty of Physics, Mathematics and Natural Sciences

educational division (faculty/institute/academy) as higher education programme developer

COURSE SYLLABUS

Functional differential equations and nonlocal boundary value problems
course title

Recommended by the Didactic Council for the Education Field of:

01.04.01 Mathematics
field of studies / speciality code and title

The course instruction is implemented within the professional education programme of higher education:

«Functional methods in differential equations and interdisciplinary research»

higher education programme profile/specialisation title

1. COURSE GOAL(s)

The purpose of mastering the discipline " Functional differential equations and nonlocal boundary value problems" is to learn the main properties and contemporary qualitative methods of study of non-classical problems for partial differential equations including elliptic equations with nonlocal boundary conditions and boundary value problems for functional differential equations

2. REQUIREMENTS FOR LEARNING OUTCOMES

Mastering the discipline " Functional differential equations and nonlocal boundary value problems " is aimed at developing the following competencies (parts of competencies):

Table 2.1. List of competences that students acquire through the course study

Code	Competence	Competence achievement indicators (within this discipline)
GC-2	He is able to determine the range of tasks within the set goal and choose the best ways to solve them, based on existing legal norms, available resources and limitations	**GC-2.1.** Formulates a project task based on the problem posed and a way to solve it through the implementation of project management**,** **GC-2.2.** Develops the concept of the project within the framework of the designated problem: formulates the goal, objectives, justifies the relevance, significance, expected results and possible areas of their application; **GC-2.3.** Plans the necessary resources, including taking into account their interchangeability; **GC-2.4**. Develops a project implementation plan using planning tools; **GC-2.5**. Monitors the progress of the project, corrects deviations, makes additional changes to the project implementation plan, clarifies the areas of responsibility of project participants
GC-3	He is able to carry out social interaction and realize his role in the team	**GC-3.1**. Develops a strategy of cooperation and on its basis organizes the selection of team members to achieve the goal; **GC-3.2.** Plans and corrects the work of the team taking into account the interests, behavioral characteristics and opinions of its members; **GC-3.3.** Resolves conflicts and contradictions in business communication on the basis of taking into account the interests of all parties **GC-3.4.** Organizes discussions on a given topic and discussion of the results of the team's work with the involvement of opponents to the developed ideas; **GC-3.5.** Organizes discussions on a given topic and discussion of the results of the team's work with the involvement of opponents to the developed ideas
PC-2	Able to mathematically correctly set natural science problems, knowledge of the formulation of classical problems of mathematics	**PC-2.1.** Conducts an initial search for information on a given topic **PC.2.2.** Analyzes and summarizes the results of the patent search on the subject of the project in the chosen field of mathematics

Code	Competence	Competence achievement indicators (within this discipline)
PC-3	Able to strictly prove the statement, formulate the result, see the consequences of the result	**PC-3.1.** Knows and can apply modern mathematical methods in practice **PC-3.2.** Able to study mathematical structure using computational methods

3.COURSE IN HIGHER EDUCATION PROGRAMME STRUCTURE

The discipline "Functional differential equations and nonlocal boundary value problems" refers to the part formed by the participants in the educational relations of block B1 of the EP HE.

As part of the EP HE, students also master other disciplines and / or practices that contribute to the achievement of the planned results of mastering the discipline "Functional differential equations and nonlocal boundary value problems".

Table 3.1. The list of the higher education programme components/disciplines that contribute to the achievement of the expected learning outcomes as the course study results

Code	Competence	Previous disciplines/modules, practices	Subsequent disciplines/modules, practices*
GC-2	He is able to determine the range of tasks within the set goal and choose the best ways to solve them, based on existing legal norms, available resources and limitations	-	State exam
GC-3	He is able to carry out social interaction and realize his role in the team	-	State exam
PC-2	Able to mathematically correctly set natural science problems, knowledge of the formulation of classical problems of mathematics	-	Interdisciplinary term paper Research work Pre-graduation practical training State exam
PC-3	Able to strictly prove the statement, formulate the result, see the consequences of the result	-	Interdisciplinary term paper Research work Pre-graduation practical training State exam

4. COURSE WORKLOAD AND ACADEMIC ACTIVITIES

The total labor intensity of the discipline " Functional differential equations and nonlocal boundary value problems " is 6 credits.

*Table 4.1. Types of academic activities during the periods of higher education programme mastering (**full-time training**)**

Type of study work		TOTAL, **a**.h.	Semester			
			1	**2**	**3**	**4**
Contact academic hours		*72*	*72*			
including:						
Lectures (LC)		36	36			
Lab work (LW)						
Seminars (workshops/tutorials) (S)		36	36			
Self-studies		*72*	*72*			
Evaluation and assessment (exam/passing/failing grade)		*36*	*36*			
Course workload	a.h.	**216**	**216**			
	credits	**6**	**6**			

5. COURSE CONTENTS

Table 5.1. Course contents and academic activities types

Course Module Title	Brief Description of the Module Content	Type of study work
Section 1. Nonlocal boundary value problems. Preamble	Functional spaces used in the course, interpolation inequalities, facts from the theory of boundary value problems for elliptic equations with a parameter, as well as boundary value problems for elliptic equations in nonsmooth domains. Origin of nonlocal boundary value problems and their classification	Lecture, seminar
Section 2. Nonlocal problems with nonlocal terms supported strictly inside the domain	Problem statement and interpretation of the nonlocal condition. Example: solving the problem for the Poisson equation in a circle by the method of separation of variables with a nonlocal condition connecting the trace of the desired function on the boundary of the circle with its trace on the inner circle. A priori estimate and solvability of an elliptic boundary value problem with a parameter in the norms of Sobolev spaces depending on a parameter. Proof of an a priori estimate in Sobolev spaces for solutions of the problem for an elliptic equation with a nonlocal boundary condition in the case where the support of nonlocal terms lies inside the domain. Fredholm solvability in Sobolev spaces, discreteness and sectorial structure of the spectrum of the nonlocal boundary value problem	Lecture, seminar
Section 3. Nonlocal problems with nonlocal terms supported near the boundary	Solution of the model Dirichlet problem for the Poisson equation in an infinite plane angle in weighted spaces. Transition to a bounded domain, localization technique. Statement of the problem for the Poisson equation with a nonlocal condition connecting the trace of a function on a part of the boundary with its trace on the manifold lying in the closure of the domain and having a nonempty intersection with the boundary. Proof of an a priori estimate for solutions in weighted spaces. Construction of a right regularizer in weighted	Lecture, seminar

	spaces for an operator of the nonlocal boundary value problem using the method for compensating of nonlocal terms	
Section 4. Nonlocal problems in the case where the support of nonlocal terms approaches the conjugation points	Solution of a model nonlocal problem for the Poisson equation in an infinite angle on a plane with a boundary condition connecting the trace of the desired function on the side of the angle with its trace on a ray inside the angle. Investigation of a nonlocal problem with a parameter on an interval, conditions for unique solvability. Statement of a nonlocal problem for an elliptic equation in a bounded plane domain in the case where the support of nonlocal terms approaches the conjugation points. Proof of the a priori estimate and construction of the right regularizer. Investigation of the asymptotics of solutions near the conjugation points	Lecture, seminar
Section 5. Boundary value problems for functional differential equations. Preamble	Variational and boundary value problems with the deviating argument. Solvability and regularity of generalized solutions. Boundary value problems for differential-difference equations in the one-dimensional case. Reduction of the boundary value problem for a differential-difference equation on an interval to a differential equation with the nonlocal boundary conditions. Second order elliptic equations in a cylinder with the nonlocal boundary conditions	Lecture, seminar
Section 6. Strongly elliptic systems of differential equations	Investigation of the Garding-type inequality for scalar equations and systems of partial differential equations. Derivation of necessary and sufficient conditions. Localization technique for the case of variable coefficients.. Comparison of the conditions for ellipticity and strong ellipticity. Solvability and spectral properties of the Dirichlet problem for a strongly elliptic system of differential equations	Lecture, seminar
Section 7. Boundary value problems for elliptic differential-difference equations	Difference operators in bounded domains of Euclidean space. Partition of the domain generated by the difference operator. Matrix description of difference operators, comparison with the symbol of the difference operator. Solution of the coercivity problem (study of the Garding-type inequality) for differential-difference operators. Obtaining sufficient conditions and necessary conditions for strong ellipticity in algebraic form. Statement of the first boundary value problem for a strongly elliptic differential-difference equation, definition of a generalized solution. Solvability and structure of the spectrum. Investigation of the smoothness of generalized solutions of the first boundary value problem for strongly elliptic differential-difference equations. Internal smoothness in subdomains. The effect of breaking the smoothness when approaching the boundary of the subdomain. Examples of maintaining smoothness in subdomains as well as in the entire domain	Lecture, seminar
Section 8. Boundary value problems for elliptic functional differential equations with the expanded and compressed arguments	Functional operators with expansion and compression of the arguments, their properties in Sobolev spaces. Description based on the Gel'fand transformation. The model boundary value problem for an elliptic functional differential equation with expanded and compressed arguments in a star-shaped domain. The effect of appearance of an infinite-dimensional kernel and cokernel. The coercivity problem for a functional differential operator with expanded and compressed arguments in a bounded domain containing the center of compressions. Obtaining an algebraic criterion for strong ellipticity in the form of the positivity of the scalar operator symbol (a combination of	Lecture, seminar

	the Fourier and Gel'fand transforms). Application to differential-difference operators. Solvability and spectrum of the first boundary value problem for a strongly elliptic functional differential equation with expansions and compressions of the arguments. Investigation of the smoothness of generalized solutions in particular cases. Singularities of generalized solutions of the first boundary value problem for a strongly elliptic equation near the origin (center of compressions)	

6. CLASSROOM EQUIPMENT AND TECHNOLOGY SUPPORT REQUIREMENTS

Table 6.1. Classroom equipment and technology support requirements

Classroom type	Classroom equipment	Specialized educational/laboratory equipment, software and materials for mastering the discipline
Lecture	An auditorium for lecture-type classes, equipped with a set of specialized furniture; board (screen) and technical means of multimedia presentations.	-
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, current control and intermediate certification, equipped with a set of specialized furniture and technical means for multimedia presentations.	-
For independent work of students	An auditorium for conducting seminar-type classes, group and individual consultations, current control and intermediate certification, equipped with a set of specialized furniture and technical means for multimedia presentations.	-

7. RESOURCES RECOMMENDED FOR COURSE STUDY

Main literature:

1. A. L. Skubachevskii, "Nonclassical boundary value problems. I", Journal of Mathematical Sciences, 155:2 (2008), 199–334.
2. A. L. Skubachevskii. Elliptic functional differential equations and applications. Basel-Boston-Berlin: Birkhauser, 1996.
3. L. E. Rossovskii, "Elliptic functional differential equations with contractions and extensions of independent variables of the unknown function", Journal of Mathematical Sciences, 223:4 (2017), 351–493.

Additional literature:

1. A. L. Skubachevskii, “Nonclassical boundary-value problems. II”, Journal of Mathematical Sciences, 166:4 (2010), 377–561.
2. A. L. Skubachevskii, “Boundary-value problems for elliptic functional-differential equations and their applications”, Russian Math. Surveys, 71:5 (2016), 801–906.
3. Antonevich A., Lebedev A. Functional-Differential Equations. I. C*-theory. Harlow: Longman, 1994.

Resources of the information and telecommunications network "Internet":

1. RUDN ELS and third-party ELS, to which university students have access on the basis of concluded agreements:
- RUDN Electronic Library System - RUDN EBS http://lib.rudn.ru/MegaPro/Web
- ELS "University Library Online" http://www.biblioclub.ru
- EBS Yurayt http://www.biblio-online.ru
- ELS "Student Consultant" www.studentlibrary.ru
- EBS "Lan" http://e.lanbook.com/
- EBS "Trinity Bridge"

2. Databases and search engines:
- electronic fund of legal and normative-technical documentation http://docs.cntd.ru/
- Yandex search engine https://www.yandex.ru/
- Google search engine https://www.google.ru/
- abstract database SCOPUS http://www.elsevierscience.ru/products/scopus/

8. ASSESSMENT TOOLKIT AND GRADING SYSTEM* FOR EVALUATION OF STUDENTS’ COMPETENCES LEVEL UPON COURSE COMPLETION

Evaluation materials and a point-rating system* for evaluating the level of formation of competencies (parts of competencies) based on the results of mastering the discipline "Non-Euclidean geometries and their applications" are presented in the Appendix to this Work Program of the discipline

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