Federal State Autonomous Educational Institution of Higher Education
Peoples' Friendship University of Russia named after Patrice Lumumba

Academy of Engineering
(name of the main educational unit (MEU) that developed the educational program of higher education)

WORKING PROGRAM OF THE DISCIPLINE

MATHEMATICS FOR SPATIAL SCIENCES
(name of discipline/module)

Recommended for the field of study/specialty:

27.04.04 CONTROL IN TECHNICAL SYSTEMS
(code and name of the field of study/specialty)

The discipline is mastered within the framework of the implementation of the main professional educational program of higher education (EP HE):

Artificial Intelligence, Machine Learning, and Space Science
(name (profile/specialization) of the educational institution of higher education)

2026

1. THE GOAL OF MASTERING THE DISCIPLINE

The course "Mathematics for Spatial Sciences" is part of the master's program "Artificial Intelligence, Machine Learning, and Space Sciences" in the 27.04.04 "Control in Technical Systems" program and is studied in the first semester of the first year. The course is offered by the department of the partner university. It consists of nine sections and 19 topics and focuses on the study of methods for the mathematical processing and analysis of spatial data.

The purpose of mastering the discipline is - the formation of a comprehensive knowledge reflecting the modern level of methods of mathematical processing and analysis of spatial data; - an expanded understanding of fundamental and modern algorithms for processing and analysis of spatial data; - the study of additional sections of matrix algebra and mathematical statistics underlying the analysis of data obtained from various sources; - the study of synthesized algorithms for optimizing the results of geodetic measurements using the least squares method.

2. REQUIREMENTS FOR THE RESULTS OF MASTERING THE DISCIPLINE

Mastering the discipline "Spatial Mathematics" aimed at developing the following competencies (parts of competencies) in students:

Table 2.1. List of competencies developed in students while mastering the discipline (results of mastering the discipline)

Cipher	Competence	Indicators of Competency Achievement (within this discipline)
GPC-1	Able to analyze and identify the natural scientific essence of control problems in technical systems based on provisions, laws and methods in the field of natural sciences and mathematics	GPC-1.1 Knows the basic laws, provisions and methods in the field of natural sciences and mathematics; GPC-1.2 Able to identify the natural scientific essence of control problems in technical systems guided by the laws and methods of natural sciences and mathematics; GPC-1.3 Proficient in tools for analyzing control problems in technical systems;
GPC-2	Able to formulate control problems in technical systems and justify methods for solving them	GPC-2.1 Knows the basic methods of solving control problems in technical systems; GPC-2.2 Able to justify methods for solving control problems in technical systems; GPC-2.3 Proficient in methods of setting control problems in technical systems;
PC-2	Able to apply modern theoretical and experimental methods for developing mathematical models of objects and processes under study in the field of aerospace systems management	PC-2.1 Knows modern theoretical and experimental methods used to develop mathematical models of studied objects and processes of professional activity; PC-2.2 Able to determine the effectiveness of the methods used to develop mathematical models of the objects and processes under study; PC-2.3 Has a command of modern theoretical and experimental methods for developing mathematical models of objects and processes of professional activity in the field of study;

3. PLACE OF THE DISCIPLINE IN THE STRUCTURE OF THE EDUCATIONAL EDUCATIONAL INSTITUTION

Discipline "Spatial Mathematics" refers to the mandatory part of block 1 “Disciplines (modules)” of the educational program of higher education.

As part of the higher education program, students also master other disciplines and/or practices that contribute to the achievement of the planned results of mastering the discipline "Spatial Mathematics".

Table 3.1. List of components of the educational program of higher education that contribute to the achievement of the planned results of mastering the discipline

Cipher	Name of competence	Previous courses/modules, practical training*	Subsequent disciplines/modules, practices*
GPC-1	Able to analyze and identify the natural scientific essence of control problems in technical systems based on provisions, laws and methods in the field of natural sciences and mathematics		Undergraduate practice / Pre-graduation practice; Geoinformation Systems and Applications;
GPC-2	Able to formulate control problems in technical systems and justify methods for solving them		Undergraduate practice / Pre-graduation practice;
PC-2	Able to apply modern theoretical and experimental methods for developing mathematical models of objects and processes under study in the field of aerospace systems management		Research work / Scientific research work (acquiring primary skills in scientific research work); Undergraduate practice / Pre-graduation practice; Research Work; Dynamics and Control of Space Systems; *Artificial Neural Networks (Deep Learning)***; *Artificial Neural Networks (Deep Learning)***; Advanced Methods of Space Flight Mechanics; *Artificial Neural Networks (Reinforcement Learning)***; Operations Research and Optimization Techniques;

* - filled in accordance with the competency matrix and the SUP EP HE
** - elective courses/practices

4. SCOPE OF THE DISCIPLINE AND TYPES OF EDUCATIONAL WORK

The total workload of the discipline "Spatial Mathematics" is 7 credit units.

Table 4.1. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**
			1
Contact work, academic hours	*68*		*68*
Lectures (LC)	34		34
Laboratory work (LW)	0		0
Practical/seminar classes (SC)	34		34
Independent work of students, academic hours	*157*		*157*
Control (exam/test with assessment), academic hours	*27*		*27*
Total complexity of the discipline	**academic hours**	***252***	*252*
	credit	***7***	*7*

5. CONTENT OF THE DISCIPLINE

Table 5.1. Content of the discipline (module) by types of academic work

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 1	Introduction. General Provisions. Objectives and tasks, areas of application of geostatistical analysis.	1.1	Basic concepts of geostatistics: random variable, spatial variable, random function.	Definitions of random variables, spatial variables, and random functions. The role of spatially referenced data. The difference between geostatistical data and classical statistical samples.	LC, SC
		1.2	Moments of spatial functions in linear geostatistics: mathematical expectation, variance, covariance, variogram. Ergodicity of random functions.	Spatial data characteristics: mean, variance, covariance, and variogram. Ergodicity of random functions and its implications for spatial data analysis.	LC, SC
		1.3	Conditions necessary for the application of geostatistical methods	Requirements for input data: stationarity, isotropy or anisotropy, and sufficient sample size. Limitations and assumptions in spatial analysis.	LC, SC
Section 2	Review of fundamental methods of mathematical processing of measurement results	2.1	Spatial data optimization algorithms using the ordinary least squares (OLS) method.	The essence of the least squares method for processing spatial data. Parametric and correlated versions of the least squares method, and their comparative characteristics.	LC, SC
		2.2	The main stages of algorithm implementation, assessment of the accuracy of the initial and optimized values of spatial data.	Sequence of steps in least-squares optimization. Accuracy assessment criteria: root-mean-square errors of the original and adjusted (optimized) values.	LC, SC
Section 3	Block matrices	3.1	Definition of block matrices.	The concept of block-represented matrices. The structure and methods of partitioning matrices into blocks as applied to spatial data.	LC, SC
		3.2	Operations with block matrices: addition, transposition, multiplication, inversion. Solution of systems of linear algebraic equations. equations (SLAE) in block notation.	Addition, transposition, multiplication, and inversion of block matrices. Solution of systems of linear algebraic equations (SLAEs) in block notation for spatial analysis problems.	LC, SC
Section 4	Mathematical processing and analysis of correlated paired data.	4.1	Using block matrices to construct algorithms for correlated and parametric versions of least squares optimization of paired data.	Application of the block approach to the equalization of paired measurements. Features of constructing algorithms for dependent (correlated) data.	LC, SC
		4.2	Analysis of paired data for the presence of rough measurements and testing the hypothesis of insignificance of the mean differences of paired data	A method for identifying anomalous (gross) errors in paired data. A statistical test of the hypothesis that the mean difference between pairs of observations is zero.	LC, SC
Section 5	Redundancy matrix of the least squares algorithm for	5.1	Determination of the redundancy matrix for parametric and correlated versions of least	The concept of a redundancy matrix, its structure and role in assessing the impact of errors in the initial data on optimization	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
	spatial data optimization and its potential.		squares data optimization.	results.	
		5.2	Post hoc analysis of the scale accuracy index using the redundancy matrix.	Post-processing accuracy assessment (posterior estimation). Calculating the scaling index (unit weight variance) using the elements of the redundancy matrix.	LC, SC
Section 6	Spatial position of a point and indicators of spatial position accuracy.	6.1	Position of a point in one-dimensional, two-dimensional, and three-dimensional space. Root mean square errors (RMSE) of the point's position in space.	Methods for describing point coordinates. Root mean square errors (RMSE) for determining a point's position in space of various dimensions.	LC, SC
		6.2	Relative position coefficients of points in space. Relative distance coefficients between points. Relative orientation coefficients of a line connecting two points in two-dimensional and three-dimensional space.	Estimating the accuracy of the distance between two points. Estimating the accuracy of the orientation (direction) of a line connecting the points in two-dimensional and three-dimensional space.	LC, SC
Section 7	A synthesized version of the correlated version of least squares optimization and spatial data analysis.	7.1	Derivation of the algorithm for the synthesized version of the correlated version. Step-by-step implementation checks.	Logic for constructing a synthesized algorithm for the least squares correlation method. Step-by-step verification of the algorithm's implementation.	LC, SC
		7.2	Finding a redundancy matrix using blocks of the inverse synthesized matrix. A posteriori assessment of data accuracy.	Obtaining a redundancy matrix from the blocks of the inverse matrix of the synthesized algorithm. A posteriori assessment of data accuracy based on the resulting matrix.	LC, SC
Section 8	A synthesized version of the parametric version of least squares optimization and spatial data analysis	8.1	Derivation of the algorithm for the synthesized version of the parametric version. Step-by-step implementation checks.	Construction of a synthesized algorithm for the parametric least squares method. Control relations at the computational stages.	LC, SC
		8.2	Finding a redundancy matrix using inverse matrix synthesis blocks. A posteriori estimation of data accuracy.	Calculating the redundancy matrix from the inverse matrix blocks for the parametric case. A posteriori assessment of spatial data accuracy.	LC, SC
Section 9	A universal synthesized algorithm for least squares optimization and spatial data analysis	9.1	Derivation of a universal synthesized algorithm. Step-by-step implementation checks. Finding the matrix redundancies using inverse matrix synthesis blocks.	Generalization of the correlation and parametric approaches in a single synthesized algorithm. Step-by-step implementation checks. Obtaining a redundancy matrix through blocks of the inverse synthesized matrix.	LC, SC
		9.2	A posteriori data accuracy assessment. Application of a universal synthesized algorithm for least-squares optimization of data in "free" geodetic constructions.	Accuracy assessment after optimization for cases where the source data does not have redundant rigid constraints (free constructions). Specifics of applying a universal algorithm to such problems.	LC, SC

* - to be completed only for FULL-TIME education: LC – lectures; LW – laboratory work; SC – practical/seminar classes.

6. LOGISTIC AND TECHNICAL SUPPORT OF DISCIPLINE

Table 6.1. Material and technical support for the discipline

Audience type	Equipment of the auditorium	Specialized educational/laboratory equipment, software and materials for mastering the discipline (if necessary)
Lecture	A lecture hall equipped with specialized furniture, a whiteboard (screen), and multimedia presentation equipment.	
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, ongoing monitoring and midterm assessment, equipped with a set of specialized furniture and technical means for multimedia presentations.	
For independent work	A classroom for independent student work (can be used for seminars and consultations), equipped with a set of specialized furniture and computers with access to the Electronic Information System.	

* - the classroom for independent work of students MUST be indicated!

7. EDUCATIONAL, METHODOLOGICAL AND INFORMATIONAL SUPPORT OF THE DISCIPLINE

Main literature:

1. Arlinghaus SL, Kerski JJ Spatial mathematics: Theory and practice through mapping. – CRC Press, 2013.
2. Pebesma E., Bivand R. Spatial data science: With applications in R. – Chapman and Hall/CRC, 2023.

Further reading:

1. Legendre G. (ed.). Mathematics of space. – John Wiley & Sons, 2011.
2. Wilf HS Mathematics for the physical sciences. – Courier Corporation, 2013

Resources of the information and telecommunications network "Internet":

1. RUDN University Electronic Library System and third-party electronic library systems to which university students have access based on concluded agreements
   - RUDN University Electronic Library System – RUDN University Electronic Library System https://mega.rudn.ru/MegaPro/Web
   - Electronic Library System "University Library Online" http://www.biblioclub.ru
   - EBS Yurayt http://www.biblio-online.ru
   - Electronic Library System "Student Consultant" www.studentlibrary.ru
   - EBS “Knowledge” https://znanium.ru/
2. Databases and search engines
   - Sage https://journals.sagepub.com/
   - Springer Nature Link https://link.springer.com/
   - Wiley Journal Database https://onlinelibrary.wiley.com/
   - Scientometric database Lens.org https://www.lens.org

Educational and methodological materials for independent work of students in mastering a dis-

cipline/module:*

1. Lecture course on the subject "Spatial Mathematics".

* - all teaching and methodological materials for independent work of students are posted in accordance with the current procedure on the discipline page in TUIS!

DEVELOPER:

Associate Professor		Saltykova Olga Alexandrovna
Position, DEPARTMENT	*Signature*	*Surname I.O.*

HEAD OF THE DEPART-MENT:

Position of the DEPARTMENT	*Signature*	*Surname I.O.*

HEAD OF THE EP HE:

Professor		Razumny Yuri Nikolaevich
Position, DEPARTMENT	*Signature*	*Surname I.O.*